

KKLT type models with moduli-mixing superpotential

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Abstract

We study KKLT type models with moduli-mixing superpotential. In several string models, gauge kinetic functions are written as linear combinations of two or more moduli fields. Their gluino condensation generates moduli-mixing superpotential. We assume one of moduli fields is frozen already around the string scale. It is found that Kähler modulus can be stabilized at a realistic value without tuning 3-form fluxes because of gluino condensation on (non-)magnetized D-brane. Furthermore, we do not need to highly tune parameters in order to realize a weak gauge coupling and a large hierarchy between the gravitino mass and the Planck scale, when there exists non-perturbative effects on D3-brane. SUSY breaking patterns in our models have a rich structure. Also, some of our models have cosmologically important implications, e.g., on the overshooting problem and the destabilization problem due to finite temperature effects as well as the gravitino problem and the moduli problem.

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1 Introduction

String/M theory is a promising candidate for unified theory including gravity. Its 4D effective theory, in general, includes many moduli fields. Their vacuum expectation values (VEVs) play an important role in particle physics and cosmology. That is because those VEVs determine coupling constants such as gauge and Yukawa couplings, and physical scales like the Planck scale M_p and the compactification scale. How to stabilize moduli (at realistic values) is one of important issues to study in string phenomenology and cosmology. How to break supersymmetry (SUSY) is another important issue. It is expected that nonperturbative effects, which fix moduli VEVs, may also break SUSY. A number of studies on these issues have been done, and some of those vacua correspond to anti-de Sitter (AdS) vacua.

Recently, flux compactification is studied intensively, because it can stabilize some of moduli. In type IIB string models, complex structure moduli and the dilaton can be stabilized around the string scale $M_{string} = \alpha'^{-1/2}$, which is expected to be of $\mathcal{O}(10^{17}) - \mathcal{O}(10^{18})$ GeV, but Kähler moduli fields are not stabilized [1]. On the other hand, in type IIA string models all of moduli can be stabilized [2], and in heterotic string models complex structure and volume moduli can be stabilized, but the dilaton VEV is not fixed. However, in the heterotic case, the compact space is generically non-Kähler, and it is mathematically difficult to treat it by present knowledge.⁴

Within the framework of flux compactification in type IIB string models, a simple model has been proposed in Ref. [4], where the remaining Kähler modulus T is stabilized by nonperturbative effects such as gluino condensation. Still, the potential minimum corresponds to the SUSY AdS vacuum at the first stage. Then, an anti-D3 ($\overline{D3}$) brane is introduced (at the tip of throat) in order to uplift the vacuum energy and realize a de Sitter (dS) or Minkowski vacuum. It shifts the potential minimum, and breaks SUSY in a controllable way. That is the KKLT scenario.

Furthermore, soft SUSY breaking terms in the KKLT scenario have been studied in Ref. [5, 6], and it has been found that in the KKLT scenario the F -term of modulus field T is of $\mathcal{O}(m_{3/2}M_p/4\pi^2)$, where $m_{3/2}$ is the gravitino mass. That is, the modulus F -term F^T and the anomaly mediation [7, 8] are comparable in soft SUSY breaking terms. That leads to a quite novel pattern of SUSY breaking terms. (See for their phenomenological aspects Ref. [9, 10, 11].) It is useful to introduce the parameter α as

⁴Recently, a solution that compact space is conformally Kähler is found in Ref. [3].

$\alpha \equiv \frac{m_{3/2}(T+\bar{T})}{F^T \ln(M_p/m_{3/2})}$ [9], in order to represent the ratio between the modulus mediation and the anomaly mediation. One of phenomenologically interesting features in the modulus-anomaly mixed mediation is the appearance of a mirage messenger scale [9], where the anomaly mediation at the GUT scale can cancel the renormalization group effect under a certain condition. The mirage messenger scale Λ_m is estimated as $\Lambda_m \sim (m_{3/2}/M_p)^{\alpha/2} M_{GUT}$. That is, soft SUSY breaking terms appear as the pure modulus mediation at the mirage scale. The mirage scale depends on the ratio between the modulus F -term and $m_{3/2}$. The original KKLT model leads to $\alpha \approx 1$, and the mirage scale is the intermediate scale.

Moreover, it has been pointed out in [12] that if the mirage scale is around $\mathcal{O}(1)$ TeV, such model has an important implication on solving the little SUSY hierarchy problem. At any rate, the mirage scale is determined by the ratio α , which has phenomenologically interesting implications.

The KKLT model is also interesting in cosmology. The anomaly mediation is sizable, that is, the gravitino mass is $\mathcal{O}(10)$ TeV. The modulus is much heavier. Thus, we may avoid the cosmological gravitino/moduli problem [6, 9, 10, 11].

In this paper, we study a modified KKLT scenario. A gauge kinetic function is a mixture of two or more moduli fields in several string models, e.g., weakly coupled heterotic string models [13, 14], heterotic M models [15, 16, 17, 18, 19], type IIA intersecting D-brane models and type IIB magnetized D-brane models [20, 21]. Suppose that the gauge kinetic function f is a linear combination of the dilaton S and the modulus T like $f = mS + wT$. Following Ref. [4], we assume that S is frozen already around M_{string} ⁵. We also assume gluino condensation generates nontrivial T -dependent superpotential, which form is expected to be $e^{-cf} \sim Be^{-bT}$. The original KKLT model corresponds to the case that $B = \mathcal{O}(1)$ and b is positive. However, in our case the constant B can be very suppressed depending on $mc\langle S \rangle$, and also the exponent coefficient b can be negative even though it is generated by asymptotically free gauge sector⁶. We study moduli stabilization and SUSY breaking with such potential terms, by adding uplifting potential. One feature of such SUSY breaking is that the ratio between $F^T/(T + \bar{T})$

⁵Alternatively, both of them may remain light in some models. We will study such models separately in Ref. [22].

⁶A similar superpotential like e^{bT} with $b > 0$ has been studied for non-asymptotically free models in Ref. [23]. However, we stress that our superpotential is generated by asymptotically free gauge sector.

and $m_{3/2}$ can vary by a value of B . That is phenomenologically interesting, because we would have richer structure of SUSY spectra. Moreover, the superpotential term with $b < 0$ has important implications on cosmology. It may avoid the overshooting problem and the destabilization problem due to finite temperature effects.

This paper is organized as follows. In section 2, we review on flux compactification and the KKLT model. In section 3, we give concrete string/M models, where gauge kinetic functions depend on two or more moduli fields. Then, in section 4 we study moduli stabilization and SUSY breaking. In section 5, we discuss implications on SUSY phenomenology and cosmology. Section 6 is devoted to conclusion. In Appendix A, detailed analysis on the potential minimum is summarized.

2 Review on flux compactification and KKLT model

2.1 Flux compactification

In this subsection, we review on flux compactification and in the next subsection we review on the KKLT model.

We consider the Type IIB $O3/O7$ orientifold 4D string model on a warped Calabi-Yau (CY) threefold with $h_{1,1}(\text{CY}) = 1$. In addition, we introduce RR and NSNS 3-form fluxes F_3^{RR} and H_3^{NS} , which should be quantized on compact 3-cycles C_3 and C'_3 such as

$$\frac{1}{2\pi\alpha'} \int_{C_3} F_3^{RR} \in 2\pi\mathbf{Z}, \quad \frac{1}{2\pi\alpha'} \int_{C'_3} H_3^{NS} \in 2\pi\mathbf{Z}. \quad (1)$$

Furthermore, these fluxes should satisfy the RR tadpole condition

$$\frac{1}{(2\pi)^4\alpha'^2} \int_{M_6} H_3^{NS} \wedge F_3^{RR} + Q^{local} = 0, \quad (2)$$

where Q^{local} is the RR charge contribution of local objects including D3-brane, wrapped D7-brane and O3-planes in the D3-brane charge unit. In this flux compactified type IIB string model, we can fix dilaton and complex structure moduli including a warp factor but not the Kähler modulus around the string scale. Thus, only the Kähler modulus remains light.

We are interested in two moduli, that is, one is the dilaton S and another is the overall Kähler modulus T , although the dilaton is frozen around M_{string} . They are given by

$$2\pi S = e^{-\phi} - ic_0, \quad 2\pi T = v_E^{2/3} - ic_4. \quad (3)$$

Here, ϕ is 10D dilaton and v_E is a volume of CY in the string unit within the Einstein frame. The 10D Einstein metric is given by the string metric $g_{MN}^{string} = e^{\phi/2} g_{MN}^E$. Thus, the CY volume in the string frame v is written by $v = e^{3\phi/2} v_E$. The axionic mode c_0 is the RR scalar and c_4 is the 4D Poincare dual of (1,1) part of 4-form RR potential. For example, these fields are related to gauge kinetic functions on D3 and non-magnetized D7 brane f_{Dp}

$$f_{D3} = S, \quad f_{D7} = T, \quad (4)$$

where $\langle Re f_{Dp} \rangle = g_{Dp}^{-2}$ and g_{Dp} is the gauge coupling on the Dp-brane. In this perturbative description, it is natural that $\langle Re S \rangle, \langle Re T \rangle = \mathcal{O}(1)$. In addition, these VEVs are related to the physical scales such as the 4D Planck scale M_p and the compactification scale M_{KK} [16, 6]

$$\frac{M_p}{M_{string}} = 2\sqrt{\pi}(\langle Re S \rangle \cdot \langle Re T^3 \rangle)^{1/4}, \quad \frac{M_{string}}{M_{KK}} = \left(\frac{\langle Re T \rangle}{\langle Re S \rangle} \right)^{1/4}, \quad (5)$$

where $M_{KK}/M_{string} \equiv \langle v^{-1/6} \rangle$. We need the condition $\langle Re T \rangle > \langle Re S \rangle$.

With the 3-form flux and orientifold planes in the compact space, the dilaton S and the complex structure moduli are frozen around M_{string} and a background metric is warped, then the metric in the 10D Einstein frame is given by [1]

$$ds_{10}^2 = \frac{1}{g_s^2 v_E} e^{2A(y)} g_{\mu\nu}^E dx^\mu dx^\nu + e^{-2A(y)} v_E^{1/3} \tilde{g}_{mn} dy^m dy^n. \quad (6)$$

Here, $g_s = e^{\langle \phi \rangle}$ is string coupling, $g_{\mu\nu}^E$ is 4D Einstein metric and \tilde{g}_{mn} is unwarped compact CY metric which is normalized as $\int d^6 y \sqrt{\det \tilde{g}_{mn}} = (2\pi\alpha'^{1/2})^6$. The y^m dependence of the warp factor $e^{2A(y)}$ on the throat is studied in [24, 25] and in generic point we have $e^{2A(y)} \sim 1$. A minimum of warp factor can be treated as complex structure deformation from CY conifold, and the warp factor on the tip of the throat $e^{A_{min}}$ can be stabilized by 3-form flux such that [1, 24],

$$a_0 \equiv \exp[(-2\pi h)/(3g_s f)] = \frac{e^{A_{min}}}{v_E^{1/6}}. \quad (7)$$

Here, f, h are given by RR and NSNS 3-form fluxes

$$2\pi f = \frac{1}{2\pi\alpha'} \int_A F_3^{RR}, \quad -2\pi h = \frac{1}{2\pi\alpha'} \int_B H_3^{NS}, \quad f, h \in \mathbf{Z}, \quad (8)$$

where A is 3-cycle and B is dual cycle of A near conifold singularity. Thus, if $h \gg g_s f$, we can produce exponentially large hierarchy in this string model, like the Randall-Sundrum model [26].

We consider moduli stabilization within the framework of 4D $N = 1$ effective supergravity. Here and hereafter, we use the unit that $M_p = 2.4 \times 10^{18} \text{ GeV} = 1$. Here we neglect the warp factor dependence of potential, because warping effects are not important in generic point of the compact CY space except for the small region on the warped throat. Hence, since moduli are bulk fields, they may not be affected by warping. The stabilization of S and the complex structure moduli U^α is as follows. The 3-form flux in the compact space M_6 generates the following superpotential, [27]

$$W_{flux} = W(S, U^\alpha) = \int_{M_6} G_3 \wedge \Omega, \quad (9)$$

where $G_3 = F_3^{RR} - 2\pi i S H_3^{NS}$ and Ω is the holomorphic 3-form on CY. With the following Kähler potential,

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) - \ln(-i \int_{M_6} \Omega \wedge \bar{\Omega}), \quad (10)$$

the scalar potential is written as

$$\begin{aligned} V &= e^K \left(D_a W \overline{D_b W} K^{a\bar{b}} - 3|W|^2 \right), \\ &= e^K \left(D_i W \overline{D_j W} K^{i\bar{j}} \right), \end{aligned} \quad (11)$$

where $D_a W = (\partial_a K)W + \partial_a W$, $K_{a\bar{b}} = \partial_a \partial_{\bar{b}} K$, a, b are summed over all moduli fields and i, j are moduli fields excluding T because of no-scale structure. The potential minimum, i.e. the F -flatness condition, is obtained as [1, 28]

$$D_S W = \frac{-1}{(S + \bar{S})} \int_{M_6} \overline{G_3} \wedge \Omega = 0, \quad (12)$$

$$D_{U^\alpha} W = \int_{M_6} G_3 \wedge \chi_\alpha = 0. \quad (13)$$

Here, we have used $\partial_{U^\alpha} \Omega = -(\partial_{U^\alpha} K) \Omega + \chi_\alpha$, where χ_α is a basis of primitive (2,1) forms.⁷ Thus, these moduli are generically stabilized at values of order unity around M_{string} , but the Kähler modulus cannot be stabilized at this

⁷The primitivity means $g^{j\bar{k}} \chi_{i\bar{j}\bar{k}} = 0$.

stage. The stabilization of the dilaton and the complex structure moduli is possible, because the degree of freedom of G_3 is equal to the number of the dilaton S and complex structure moduli, that is $2(1 + h_{2,1})$. From (12) and (13), the 3-form flux G_3 can be decomposed to $(0, 3)$ form $G_{(0,3)}$ and primitive $(2, 1)$ forms $G_{(2,1)}^P$. As a result, G_3 is imaginary self-dual (ISD), such that $*_6 G_3 = iG_3$. That leads to

$$\begin{aligned} (2\pi)^4 \alpha'^2 N_{\text{flux}} &\equiv \int_{M_6} H_3^{NS} \wedge F_3^{RR} = \frac{ie^\phi}{2} \int_{M_6} G_3 \wedge \overline{G}_3, \\ &= \frac{e^\phi}{2 \cdot 3!} \int_{M_6} d^6 y \sqrt{g_6} G_{mnl} \overline{G}_3^{mnl} \geq 0, \end{aligned} \quad (14)$$

then from (2) we can see that we actually need negative charge contributions, such as O3-plane, wrapped D7-brane or $\overline{\text{D3}}$ -brane etc. These ISD fluxes stabilize the position of D7-branes and $\overline{\text{D3}}$ -branes [29], when the back reaction from $\overline{\text{D3}}$ -branes to the background is neglected [30]. The other open string moduli of D7-branes cannot be stabilized in the ISD flux background. SUSY breaking effects (including imaginary anti self-dual fluxes) or other bulk geometry may stabilize them.

2.2 KKLT model

In this subsection, we review on the KKLT model. See Appendix A for details of potential analysis. We assume that the dilaton and complex structure moduli are frozen around M_{string} through flux compactification as mentioned in the previous subsection. Here, for simplicity we neglect constant Kähler potential of frozen dilaton and complex structure moduli because those only change overall magnitude of scalar potential. To stabilize the remaining Kähler modulus, the T -dependent superpotential is added in Ref. [4]. Such superpotential can be generated by non-perturbative effects at a low energy scale in the ISD flux background with $G_{(0,3)} \neq 0$. Then Kähler potential and superpotential are written as

$$K = -3 \ln(T + \overline{T}), \quad (15)$$

$$W = w_0 - C e^{-aT}, \quad (16)$$

where $a = 8\pi^2/N$ with $N \in \mathbf{N}$ and ,

$$w_0 = \langle W_{\text{flux}} \rangle = \langle \int_M G_{(0,3)} \wedge \Omega \rangle. \quad (17)$$

The second term in R.H.S. of (16) originates from $SU(N)$ ($N > 1$) gauge group gluino condensation on non-magnetized D7-branes wrapping unwarped 4-cycle or Euclidean D3-brane instanton ($N = 1$) wrapping a similar cycle. The constant C can depend on complex structure moduli, i.e. $C = C(\langle U^\alpha \rangle)$. For gluino condensation, we have $C = NM_{string}^3$ and for D3-brane instanton C is 1-loop determinant of D3-brane mode which depend on complex structure moduli. Thus, the natural order of C is of $\mathcal{O}(1)$ or suppressed by one-loop factor. This superpotential generates the following scalar potential,

$$V_F = e^K \left[|D_T W|^2 \frac{(T + \bar{T})^2}{3} - 3|W|^2 \right], \quad (18)$$

and stabilizes the Kähler modulus, such that $\partial_T V_F = D_T W = 0$, i.e.,

$$D_T W = \frac{-3}{(T + \bar{T})} \left[w_0 - \left(1 + \frac{a(T + \bar{T})}{3} \right) C e^{-aT} \right] = 0. \quad (19)$$

That corresponds to the SUSY AdS vacuum

$$\langle V_F \rangle = -3 \langle e^K |W|^2 \rangle. \quad (20)$$

Then, we require $|w_0| \ll |C|$ and the VEV of T is obtained as

$$a \text{Re} T = \ln \left[\frac{|C|(3 + a(T + \bar{T}))}{3|w_0|} \right] \simeq |\ln(w_0)| \simeq \ln \left[\frac{M_p}{m_{3/2}} \right], \quad (21)$$

$$a \text{Im} T = -\text{Arg}(w_0) + \text{Arg}(C) + 2\pi n, \quad n \in \mathbf{Z}, \quad (22)$$

that is, $a \text{Re} T = \mathcal{O}(4\pi^2)$. The gravitino mass $m_{3/2}$ and the modulus mass m_T are obtained as

$$m_{3/2}^2 \equiv \langle e^K |W|^2 \rangle = e^K \frac{a^2(T + \bar{T})^2}{9} |C|^2 e^{-2aT} \simeq |w_0|^2 \ll 1, \quad (23)$$

$$m_T \simeq a(T + \bar{T}) m_{3/2}. \quad (24)$$

In order to obtain suppressed gravitino mass, we need a very small value of w_0 , that is, $\langle |W_{flux}| \rangle \ll 1$. Study on landscape of flux vacua suggests that the number of the vacua N_{vac} with gravitino mass $m_{3/2}$ is typically given by $N_{vac} \sim m_{3/2}^2 \cdot 10^{300}$ [31, 32]. This number is amazingly large, and we can tune fluxes such that $|w_0| \ll 1$. For example [6], it is tuned as $|G_{(0,3)}/G_{(2,1)}| \ll 1$.

To realize a dS or Minkowski vacuum, we need to uplift the potential by $3m_{3/2}^2 \sim |w_0|^2 \ll 1$. That has been done in Ref. [4] by adding a single $\overline{\text{D3}}$ -brane stabilized on the tip of warped throat as an origin of uplifting scalar potential. The following potential

$$V_L = 2 \frac{a_0^4 T_3}{g_s^4} \frac{1}{4\pi^2 (ReT)^2} \equiv \frac{D}{(T + \overline{T})^2}, \quad (25)$$

where $T_3 = (2\pi)^{-3}(\alpha')^{-2}$ is D3-brane tension, is generated from $\overline{\text{D3}}$ tension and Wess-Zumino term. Because of the warp factor a_0 , the uplifting potential V_L can be very small. Then the total scalar potential is written as

$$V = V_F + V_L. \quad (26)$$

In order to have $\langle V \rangle = 0$, we have to tune the warp factor using fluxes, such as $a_0 = \exp[-2\pi h/(3g_s f)] \sim \sqrt{m_{3/2}}$.

The uplifting potential shifts the minimum. The VEV of T slightly changes and non-vanishing F^T is generated. From the above superpotential, we can estimate $W_{TT} = -aW_T$. Then, by use of analysis in Appendix A, we evaluate F^T as

$$\left| \frac{F^T}{(T + \overline{T})} \right| \simeq \frac{m_{3/2}}{a ReT}, \quad (27)$$

$$Arg(\langle F^T \rangle) = Arg(\langle \overline{W} \rangle) = -Arg(w_0). \quad (28)$$

It is useful to use the following parameter α ,

$$\alpha \equiv \frac{m_{3/2}(T + \overline{T})}{F^T \ln(M_p/m_{3/2})}, \quad (29)$$

in order to represent the ratio between anomaly mediation and modulus mediation. In this case we have $\alpha \simeq 1$. We can delete phase of superpotential due to $U(1)_R$ symmetry $W \rightarrow e^{-iArg(w_0)}W$ and PQ symmetry $aT \rightarrow aT + i(Arg(C) - Arg(w_0))$ [6, 33]. The results of (23) and (24) do not change.

We show an illustrating example in Figure 1. We take parameters as $C = N$, $a = 8\pi^2/N$, $N = 5$ and $D = 6.3 \times 10^{-27}$. Then, we obtain $\langle ReT \rangle \simeq 2.2$ and $m_{3/2} \simeq 25$ TeV. A height of bump at $ReT \sim 2.4$ in this example is $3m_{3/2}^2$, because we uplifted the potential by $3m_{3/2}^2$. In general, the height of bump is estimated as $\mathcal{O}(m_{3/2}^2)$.

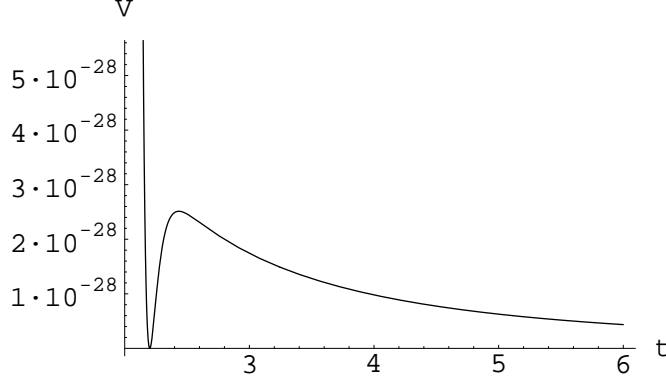


Figure 1: A potential plot of KKLT model with parameters, $w_0 = 10^{-13}$, $C = N$, $a = 8\pi^2/N$, $N = 5$ and $D = 6.3 \times 10^{-27}$. Then, we obtain $\langle ReT \rangle \simeq 2.2$ and $m_{3/2} \simeq 25$ TeV. A horizontal axis is $t = ReT$ and a vertical axis is V .

3 Moduli mixing in gauge coupling

In several string models, the gauge kinetic function is obtained as a linear combination of two or more moduli fields. In this section we show concrete examples.

First, in heterotic (M-)theory one-loop gauge coupling is given by [18]

$$f_{strong} = S - \beta T + f_{M5},$$

$$\beta \sim \frac{1}{16\pi^4} \int_{CY} J \wedge \left[\text{Tr}(F^{(2)})^2 - \frac{1}{2} \text{Tr}(R^2) \right], \quad (30)$$

where f_{strong} is the gauge kinetic function of a strong gauge group and J is the Kähler form on CY with $h_{1,1} = 1$. This can be seen from 10D Green-Schwarz term $\int_{M_{10}} B_2 \wedge X_8$ or 11D Chern-Simmons term $\int_{M_{11}} C_3 \wedge G_4 \wedge G_4$. The third term f_{M5} denotes a contribution from M5-brane position moduli Y in the orbifold interval, such as $f_{M5} = \alpha Y^2/T$, $\alpha \sim \int_{CY} J \wedge *_6 J$. Hence, gluino condensation may generate moduli mixing superpotential.

$$W_{GC} \sim \exp[-a f_{strong}], \quad a = 8\pi^2/N \quad \text{for } SU(N). \quad (31)$$

That is, the fact that moduli mix in the gauge couplings, implies that moduli may mix in non-perturbative superpotential.⁸

⁸Moreover, M2-brane stretched between M5-brane and strong coupling M9-brane can

Type II models such as intersecting D-brane models or magnetized D-brane models have gauge couplings similar to those in heterotic models [21]. For example, in supersymmetric type IIB magnetized D-brane models on $T^6/(Z_2 \times Z_2)$ orbifold with $h_{1,1}^{bulk} = 1$ ⁹, gauge couplings are given as follows,

$$\begin{aligned} f_{mD7} &= |m_7|S + |w_7|T, \\ f_{mD9} &= m_9S - w_9T \quad \text{for } O3/O7 \text{ system,} \end{aligned} \quad (32)$$

where m_p, w_p ($p = 7, 9$) $\in \mathbf{Z}$ are Abelian magnetic flux contribution F from world volume and Wess-Zumino term. In this case, Abelian gauge magnetic flux F is quantized on a compact 2-cycle C_2 as $\int_{C_2} F \in \mathbf{Z}$. Then, m_p ($p = 7, 9$) is given by $m_7 = \int_{mD7} F \wedge F$ and $m_9 = \int_{mD9} F \wedge F \wedge F$. On the other hand, w_9 is the winding number on a wrapping 4-cycle and magnetic flux contribution, $w_9 = \int_{mD9} *_6 J_{bulk} \wedge F$ up to a numerical factor. Moreover, w_7 is the winding number of D7-brane on the 4-cycle. Thus, signs of m_9 and w_9 can depend on magnetic fluxes and SUSY conditions. For example in [35], one can find negative m_9 and w_9 . In addition T-duality action can exchange the winding number for the magnetic number, but the result is similar, that is

$$f_{mD9} = W_9S - M_9T \quad \text{for } O9/O5 \text{ system,} \quad (33)$$

where $W_9, M_9 \in \mathbf{Z}$, W_9 is the winding number on the 6-cycle and M_9 is the winding number on the 2-cycle and magnetic flux contributions, $M_9 = \int_{mD9} J_{bulk} \wedge F \wedge F$. Here we have neglected numerical factors again.

The gauge coupling on these magnetized brane is written by

$$\frac{1}{g_{mD9}^2} = |m_9 ReS - w_9 ReT|. \quad (34)$$

generates [34]

$$W_{M2} \sim \exp [-(\beta T - Y)].$$

This is also moduli mixing superpotential.

⁹Actually, $T^6/(Z_2 \times Z_2)$ orbifold, whose orbifold twists are

$$\begin{aligned} \theta &: (z^1, z^2, z^3) \rightarrow (-z^1, -z^2, z^3) \\ \omega &: (z^1, z^2, z^3) \rightarrow (z^1, -z^2, -z^3), \end{aligned}$$

have three Kähler forms in the bulk, that is $h_{1,1}^{bulk} = 3$. We identify indices of those cycles for simplicity.

Note that magnetic fluxes can contribute to RR tadpole condition of 4-form potential and 8-form potential [35] [36] and they should satisfy the tadpole cancellation condition,

$$\begin{aligned} \frac{N_{\text{flux}}}{2} + \sum_{p=7,9} \sum_{a=\text{stacks}} N_p^a m_p^a + Q_3^{\text{others}} &= 0, \\ \sum_{p=7,9} \sum_{a=\text{stacks}} N_p^a w_p^a + Q_7^{\text{others}} &= 0, \end{aligned} \quad (35)$$

where Q_p^{others} ($p = 3, 7$) are contributions of non-magnetized Dp-brane and Op-plane and N_p^a is the stack number of magnetized Dp-branes. In this paper, we treat w_p , m_p as free parameters because we only concentrate on a stack of magnetized D-branes.

In Type IIA intersecting D-brane models, which are T-duals of the above IIB string models, the above expressions of Kähler moduli change for complex structure moduli. However, since there are 3-form and even-form fluxes, all geometric moduli can be frozen in this type IIA model at low energy as a supersymmetric AdS vacuum.

In orbifold string theory, moduli in twisted sector, the so-called twisted moduli M , can exist [37]. These modes can contribute to gauge kinetic function on D-brane near orbifold fixed point,

$$f_{Dp} = (S \text{ or } T) + \sigma M, \quad (36)$$

where σ is $\mathcal{O}(0.1) - \mathcal{O}(1)$ of parameter, depending on gauge and orbifold group. These twisted moduli may stabilize easily due to their Kähler potential [38], but make little contribution to gauge coupling because the moduli are related to orbifold collapsed cycle. Then, we may naturally have $\langle M \rangle \ll 1$.

4 KKLT models with moduli-mixing superpotential

As seen in the previous section, gauge kinetic functions are linear combinations of two or more moduli fields in several string models. Thus, in this section we study the model that the gauge kinetic function for gluino condensation superpotential is a linear combination of two moduli, say S and T , but one of them, S , is frozen around M_{string} , e.g. by flux compactification.¹⁰

¹⁰In Ref. [22], we will study the model that both of S and T remain light.

For concreteness, we use the terminology of models based on O3/O7 type IIB string theory, in particular the gauge kinetic functions (32). However, if we can realize the above situation in other string models, the following discussions are applicable for such string models.

After dilaton and complex structure moduli are frozen out in the ISD flux background around M_{string} , we consider the following Kähler potential and superpotential at low energy,

$$K = -n_S \ln(\langle S \rangle + \langle \bar{S} \rangle) - n_T \ln(T + \bar{T}), \quad n_S, n_T \in \mathbf{N}, \quad (37)$$

$$W(T) = w(T) \pm B e^{\pm bT}, \quad b > 0. \quad (38)$$

For the first term $w(T)$ in R.H.S. of (38), we study the following three cases

$$w(T) = \langle \int G_3 \wedge \Omega \rangle, \quad (39)$$

$$w(T) = e^{-\frac{8\pi^2 \langle S \rangle}{N_a}}, \quad (40)$$

or

$$w(T) = A e^{-\frac{8\pi^2 T}{N_a}}. \quad (41)$$

In the second case, the exponential term of $\langle S \rangle$ can be generated by gluino condensation on a stack of N_a D3-branes which is far from warped throat¹¹. At any rate, the first and second cases correspond to $w(T) = w_0 = \text{constant}$. Thus, their potential analysis is the almost same. The third case can be realized on a stack of N_a non-magnetized D7-branes or Euclidean D3-brane which is far from the throat. Alternatively, this term can be generated by magnetized D7-branes, and in this case we have $A = e^{-8\pi^2 \langle S \rangle / N_a}$. Next, we explain the second term $B e^{\pm bT}$ in (38). Since gauge coupling on magnetized D-brane is given by $f_b \equiv m_b S \pm w_b T$ ($m_b, w_b \in \mathbf{N}$) like (32), strong dynamics on those branes may generate a term like e^{-cf} . Thus, the term e^{-bT} ($b > 0$) means that the superpotential can be generated by gluino condensation on (non-)magnetized D7-brane by (32). On the other hand, the term e^{bT} can imply gluino condensation on magnetized D9-brane and we may use this potential so far as a gauge coupling is weak, i.e.

$$m_b \text{Re} S > w_b \text{Re} T. \quad (42)$$

¹¹If we have another inflationary warped throat, such as $a'_0 \sim 10^{-3}$, gluino condensation on a stack of N D7-branes or D3-brane on the tip of that throat can also generate a $\langle S \rangle$ dependent superpotential $w_0 \sim (a'_0)^3 e^{-\frac{8\pi^2 \langle S \rangle}{N}}$.

In this case from (5) and (42), we need the condition $\frac{m_b}{w_b} > \frac{\langle ReT \rangle}{\langle ReS \rangle} > 1$. Now, since the dilaton S is stabilized at very high energy because of 3-form fluxes, then we can write $f = m_b S \pm w_b T \rightarrow m_b \langle S \rangle \pm w_b T$. Thus, at low energy, gluino condensation on these magnetized D-branes can generate a very suppressed value of $|B|$, which is given by

$$B \equiv C e^{-m_b c \langle S \rangle} \quad , \quad c = \frac{8\pi^2}{N_b} \quad \text{for } SU(N_b), \quad (43)$$

i.e., $|B| \ll 1$. We assume that C is of $\mathcal{O}(1)$ and one may find $b = c w_b$.

Then, at the final stage we add the following uplifting potential,

$$V_L = \frac{D}{(T + \bar{T})^{n_p}}, \quad (44)$$

and the total potential is written as $V = V_F + V_L$. We tune it such that $V = V_F + V_L = 0$. This uplifting potential for $n_p = 2$ may be induced by adding a few $\overline{D3}$ -branes at the tip of the throat. If there is a magnetized D9-brane, presence of a few $\overline{D3}$ -branes may give non-trivial effects to the D9-brane. In such case, we assume that the compact space is orbifolded, D3-branes are located at the orbifold fixed points in the bulk and $\overline{D3}$ -branes are located at the fixed point on the tip of a warped throat like [39]. Then the position of D3-branes are fixed and mass of open string tachyon may be nearly a TeV scale. Hence it may not affect to our model at this scale. In this case, we can have exotic matter on $\overline{D3}$ -brane at a TeV scale. Furthermore, we neglect twisted moduli contribution to gauge coupling as $\langle M \rangle \ll 1$. We also assume that we have a larger amount of total number of D3-branes and D7-branes in the bulk than D9-branes, in order not to change the geometry of [1].

Following [6], we consider arbitrary integer of n_p to study generic case. At any rate, our models are well-defined as 4D effective supergravity models. In what follows, we study four types of models corresponding to all of possibilities mentioned above.

4.1 Model 1

We study the following superpotential,

$$W = w_0 - B e^{-bT}. \quad (45)$$

In this model, the second term is generated by gluino condensation on a magnetized D7-brane, where a gauge kinetic function is $f = m_b \langle S \rangle + w_b T$.

If $|B| \gg |w_0|$, then this is similar to the previous KKLT model. Thus, the modulus T is stabilized at,

$$bReT \simeq \ln \left[\frac{|B|}{|w_0|} \right] \simeq -\frac{8\pi^2 m_b}{N_b} \langle S \rangle - \ln |w_0|. \quad (46)$$

Therefore, we obtain

$$\frac{8\pi^2}{N_b} \langle Ref \rangle \simeq \ln \left[\frac{M_p}{m_{3/2}} \right], \quad (47)$$

where we have used $B = Ce^{-8\pi^2 m_b \langle S \rangle / N_b}$, $C = N_b$, $b = 8\pi^2 w_b / N_b$, and the gravitino mass is obtained as $m_{3/2} \simeq w_0$.

When we add the uplifting potential (44) and tune it such that $V = V_F + V_L = 0$, then SUSY is broken and F^T is induced as

$$\frac{F^T}{(T + \bar{T})} \simeq n_p \frac{3}{n_T} \frac{m_{3/2}}{b(T + \bar{T})} e^{-i\theta_W}. \quad (48)$$

Thus, compared with the results in subsection 2.2, F^T becomes larger by the factor $\ln \frac{M_p}{|B|} = \frac{8\pi^2 m_b \langle S \rangle}{N_b}$, and the modulus mass becomes lighter by the factor $\ln \frac{|B|}{M_p}$, i.e., $m_T = b(T + \bar{T})m_{3/2}$. Thus, we obtain

$$\alpha \simeq \frac{2n_T}{3n_p \left(\frac{m_b \langle ReS \rangle}{w_b \langle ReT \rangle} + 1 \right)}. \quad (49)$$

With $n_T = 3$, $n_p = 2$, we have $0 < \alpha < 1$. Hence, the modulus mediation and anomaly mediation are still comparable except the case with $|\alpha| \ll 1$, where the modulus mediation is dominant.

When $|B|(n_T + b \langle T + \bar{T} \rangle) \leq n_T |w_0|$, this analysis may not be reliable in perturbative description, because of $\langle ReT \rangle \leq 0$. We may need quantum or α' correction to Kähler potential [40]. However, for simplicity, we do not consider such case here.

When the constant term w_0 is generated by fluxes, we have to fine-tune fluxes as $G_{(0,3)}/G_{(2,1)} \sim 10^{-13}$, in order to realize soft masses at the weak scale. On the other hand, when the constant w_0 is generated as $w_0 = e^{-8\pi^2 \langle S \rangle / N_a}$, we do not need such fine-tuning for $\langle S \rangle = \mathcal{O}(1)$. (See also Ref. [41].)

Generic form of potential in this model is similar to Figure 1. The height of the bump is of $\mathcal{O}(m_{3/2}^2)$, and this potential has the runaway behavior at the right of the bump.

4.2 Model 2

We study the following superpotential,

$$W = Ae^{-aT} - Be^{-bT}. \quad (50)$$

This superpotential can be generated by gluino condensation on magnetized D7-brane and non-magnetized D7-brane. We assume that $G_{(0,3)} = 0$, then $w_0 = 0$ for simplicity. However, the following results can change quantitatively but not qualitatively for the case with $w_0 \neq 0$ [6]. This model is called the racetrack model [42]. For $aReT$, $bReT = \mathcal{O}(4\pi^2)$, the SUSY vacuum leads to

$$\frac{Ae^{-aT}}{Be^{-bT}} \simeq \frac{b}{a} = \frac{w_b N_a}{N_b} \in \mathbf{R}, \quad (51)$$

$$\begin{aligned} ReT &\simeq \frac{1}{a-b} \ln \left[\frac{|A|a}{|B|b} \right], \\ &\simeq \frac{m_b N_a}{N_b - w_b N_a} \langle ReS \rangle, \end{aligned} \quad (52)$$

$$\begin{aligned} ImT &= \frac{Arg(A) - Arg(B)}{a-b} + \frac{2\pi}{a-b} n, \quad n \in \mathbf{R}, \\ &= -\frac{m_b N_a}{N_b - w_b N_a} \langle ImS \rangle + \frac{N_a N_b}{2(N_b - w_b N_a)} n, \end{aligned} \quad (53)$$

$$m_T \simeq \frac{ab(T + \bar{T})^2}{n_T} m_{3/2}. \quad (54)$$

Here, for simplicity, we have assumed that $ReS \sim 1$, $a = 8\pi^2/N_a$, $b = 8\pi^2 w_b/N_b$, $A = N_a$, $B = N_b e^{-bm_b \langle S \rangle/w_b}$ and $w_b = \mathcal{O}(1)$. From (5) we can find the physical scales

$$\frac{M_p}{M_{string}} = 2\sqrt{\pi} \langle ReS \rangle \left(\frac{N_a m_b}{N_b - w_b N_a} \right)^{3/4}, \quad \frac{M_{string}}{M_{KK}} = \left(\frac{N_a m_b}{N_b - w_b N_a} \right)^{1/4}.$$

We require the condition that the compactification scale is smaller than the string scale, i.e.,

$$w_b N_a < N_b < (m_b + w_b) N_a. \quad (55)$$

Then, we obtain

$$\frac{W_{TT}}{W_T} \simeq -ab \frac{(T + \bar{T})}{n_T}, \quad (56)$$

$$\langle Ref \rangle = m_b \langle ReS \rangle + w_b \langle ReT \rangle \simeq \frac{m_b N_b}{N_b - w_b N_a} \langle ReS \rangle. \quad (57)$$

The gravitino mass is estimated as

$$m_{3/2} \simeq e^{-a\langle ReT \rangle}. \quad (58)$$

When we add the uplifting potential (44) and tune it such that $V = V_F + V_L = 0$, then SUSY is broken. Generic form of potential is similar to Figure 1. By generic analysis in Appendix A, we obtain

$$\left| \frac{F^T}{(T + \bar{T})} \right| \simeq 3n_p \frac{m_{3/2}}{ab(T + \bar{T})^2}. \quad (59)$$

Since we have

$$\ln \left[\frac{M_p}{m_{3/2}} \right] \simeq a\langle ReT \rangle = \frac{8\pi^2}{N_b} \langle Ref \rangle = \frac{8\pi^2 m_b}{N_b - w_b N_a} \langle ReS \rangle, \quad (60)$$

we estimate the ratio parameter,

$$\alpha \simeq \frac{2}{n_p} \frac{b(T + \bar{T})}{3} \sim \mathcal{O}(4\pi^2). \quad (61)$$

That implies that anomaly mediation is dominant in SUSY breaking. These results have already been pointed out in [6], but in our model we do not need to tune fluxes in order to obtain $ReT \sim 1$ so far as satisfying (55) and $ReS \sim 1$. Furthermore, since in this case we have

$$|W| \sim |C| \exp \left[-\frac{8\pi^2 m_b}{N_b - w_b N_a} \langle ReS \rangle \right], \quad (62)$$

a very small scale may be generated by a value of ReS . That is different from the usual racetrack model.

When we consider the case that both terms in the superpotential are generated by magnetized D7-branes, and replace as $A \rightarrow N_a e^{-8\pi^2 m_a \langle S \rangle / N_a}$ with $a = 8\pi^2 w_a / N_a$, then we obtain

$$\langle ReT \rangle \simeq \frac{(N_a m_b - N_b m_a)}{(w_a N_b - w_b N_a)} \langle ReS \rangle, \quad (63)$$

and

$$\begin{aligned} \langle Ref_a \rangle &= m_a \langle ReS \rangle + w_a \langle ReT \rangle = N_a \cdot \frac{(w_a m_b - m_a w_b)}{(w_a N_b - w_b N_a)} \langle ReS \rangle, \\ &= \langle Ref_b \rangle \cdot \frac{N_a}{N_b}. \end{aligned} \quad (64)$$

In this case, the gravitino mass is estimated as

$$m_{3/2} \simeq e^{-8\pi^2 \langle Re f_a \rangle / N_a}, \quad (65)$$

and the parameter α is estimated as

$$\alpha \simeq \frac{\frac{16\pi^2}{N_b} w_b (T + \bar{T})}{3n_p \left(\frac{m_a \langle Re S \rangle}{w_a \langle Re T \rangle} + 1 \right)}. \quad (66)$$

Its natural order is of $\mathcal{O}(4\pi^2)$. That is, the anomaly mediation is dominant. In a special parameter region, we may have $\alpha = \mathcal{O}(1)$, where the modulus mediation and anomaly mediation are comparable.

4.3 Model 3

We study the following superpotential,

$$W = w_0 + B e^{bT}. \quad (67)$$

Now we consider the case that the gauge kinetic function is written by $f_b = m_b \langle S \rangle - w_b T$, and the condition of (42) should be satisfied. In this case, we assume the presence of gluino condensation on a magnetized D9-brane. For $b Re T \gg 1$, the modulus T is stabilized at

$$b Re T = \ln \left[\frac{|w_0| n_T}{|B| (b(T + \bar{T}) - n_T)} \right] \simeq \ln \left[\frac{|w_0|}{|B|} \right], \quad (68)$$

$$b Im T = Arg(w_0) - Arg(B), \quad (69)$$

where we have used $b = 8\pi^2 w_b / N_b$, $B = C e^{-b m_b \langle S \rangle / w_b}$, $C = N_b$. One might think that for $|w_0| \gg |B|$, we do not need to tune fluxes to realize $Re T \sim 1$ when $m_b \sim w_b$ and $Re S \sim 1$. However, in order to obtain a weak coupling on the magnetized D-brane at the cut-off scale, we need the following condition,

$$\langle Re f_b \rangle = m_b \langle Re S \rangle - w_b Re T \simeq -\frac{N_b}{8\pi^2} \ln |w_0| \sim 1. \quad (70)$$

Then, we must tune the parameter as $|w_0| \sim \exp[-8\pi^2 / N_b]$. In the case with $w_0 = \langle \int G_3 \wedge \Omega \rangle$, we have to fine-tune fluxes. However for $w_0 = e^{-8\pi^2 \langle S \rangle / N_a}$ with $G_{(0,3)} = 0$, we do not need such fine-tuning. In this model, since moduli

can be stabilized as $w_0 \sim \langle Be^{bT} \rangle$, the gravitino mass is given by $m_{3/2} \sim |w_0|$. For $m_b > 1$, we can find

$$\frac{8\pi^2}{N_b} \langle Ref_b \rangle \simeq \ln \left[\frac{M_p}{m_{3/2}} \right], \quad (71)$$

from (70).

When we add the uplifting potential (44) and tune it such that $V = V_F + V_L = 0$, SUSY is broken and F^T is induced. Since $W_{TT} = +bW_T$ we obtain

$$\frac{F^T}{(T + \bar{T})} \simeq -n_p \frac{3}{n_T} \frac{m_{3/2}}{b(T + \bar{T})} e^{-i\theta_W}, \quad (72)$$

$$\alpha \simeq -\frac{2n_T w_b ReT}{3n_p \langle Ref_b \rangle} = -\frac{2n_T}{3n_p \left(\frac{m_b \langle ReS \rangle}{w_b \langle ReT \rangle} - 1 \right)}. \quad (73)$$

That is, the ratio parameter α is negative. Naturally we would have $|\alpha| = \mathcal{O}(1)$.

This potential does not seem to have the runaway behavior. That has important implications on cosmology as will be discussed in the next section. These properties are remarkably different from the KKLT model, although this model only changes to $w_b \rightarrow -w_b$ from model 1.

4.4 Model 4

We study the following superpotential,

$$W = Ae^{-aT} + Be^{bT}. \quad (74)$$

In this model, we assume that non-perturbative effects on the magnetized D9-brane and the non-magnetized D7-brane generate the above superpotential. We also assume vanishing 3-form flux $G_{(0,3)} = 0$. In this model, the condition (42) should be satisfied. This superpotential can be obtained also in heterotic M-theory [43], where the first term can be originated from membrane instanton and the second term is originated from gluino condensation on the strong coupled fixed plane¹². In this heterotic model the orbifold interval T can be stabilized, because of signs of exponents.

¹²See also [44].

This type IIB model is the same as the model 2 except using magnetized D9-brane. For $aReT, bReT \gg 1$, the modulus T is stabilized as

$$ReT \simeq \frac{1}{a+b} \ln \left[\frac{|A|a}{|B|b} \right] \simeq \frac{N_a m_b}{N_b + w_b N_a} \langle ReS \rangle, \quad (75)$$

$$ImT = \frac{Arg(A) - Arg(B)}{a+b} = -\frac{N_a m_b}{N_b + w_b N_a} \langle ImS \rangle, \quad (76)$$

where we have assumed the same parameters as model 2. One may find that ReT tends to be smaller than model 2, and we may need large magnetic fluxes m_b . From (5) we can find the physical scales

$$\frac{M_p}{M_{string}} = 2\sqrt{\pi} \langle ReS \rangle \left(\frac{N_a m_b}{N_b + w_b N_a} \right)^{3/4}, \quad \frac{M_{string}}{M_{KK}} = \left(\frac{N_a m_b}{N_b + w_b N_a} \right)^{1/4}.$$

Thus, we require the following condition like model 2,

$$0 < N_b < N_a(m_b - w_b). \quad (77)$$

We have the gauge kinetic function on the magnetized D9-brane

$$\langle Ref \rangle = \frac{m_b N_b}{N_b + w_b N_a} \langle ReS \rangle. \quad (78)$$

Hence, the gauge coupling on the magnetized D9-brane can be weak so far as satisfying (77) and $\langle ReS \rangle \sim 1$. Since the dynamical scale on this D-brane is obtained as $|C| \exp[-8\pi^2 m_b \langle ReS \rangle / (N_b + w_b N_a)] \ll 1$, we can generate a small scale as model 2. Furthermore, without fine-tuning, we can have moderate values of $\langle ReT \rangle$ and the gauge coupling so far as satisfying (77). Other properties are obtained from model 2 by replacing $w_b \rightarrow -w_b$. For example we have negative α .

When we replace $A \rightarrow N_a e^{-8\pi^2 m_a \langle S \rangle / N_a}$ with $a = 8\pi^2 w_a / N_a$, results are also obtained from model 2 by replacing $w_b \rightarrow -w_b$. For example, we obtain the parameter α ,

$$\alpha \simeq -\frac{\frac{16\pi^2}{N_b} w_b (T + \bar{T})}{3n_p \left(\frac{m_a \langle ReS \rangle}{w_a \langle ReT \rangle} - 1 \right)}. \quad (79)$$

It is negative and its natural order is $\mathcal{O}(4\pi^2)$.

Here we show an example of potential of model 4 in Figure 2. It is quite different from model 1 and model 2, while in model 3 a similar potential is obtained. That has cosmologically important implications, as will be discussed in the next section.

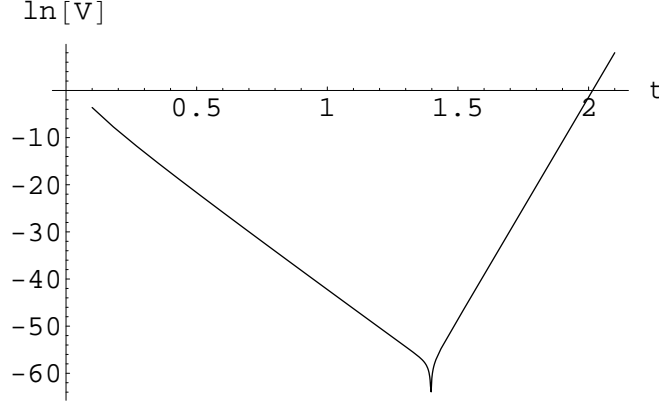


Figure 2: The potential of model 4 with parameters, $n_T = 3$, $n_S = 1$, $n_p = 2$, $N_a = 4$, $N_b = 5$, $m_b = 6$, $w_b = 3$, $\langle ReS \rangle = 1$, $D = 2.2 \times 10^{-27}$. Then, we obtain $\langle ReT \rangle \simeq 1.4$, $M_{string}/M_p \simeq 0.22$ and $m_{3/2} \simeq 40$ TeV. A horizontal axis is $t = ReT$ and a vertical axis is $\ln[V]$. This potential can make sense until $ReT < m_b \langle ReS \rangle / w_b = 2$.

5 Implications on SUSY phenomenology and cosmology

In this section, we discuss implications of our results on SUSY phenomenology and cosmology.

Concerned about SUSY breaking, our results are qualitatively similar to those in Ref. [6], that is, in model 1 and 3, the modulus mediation and anomaly mediation are comparable, while in model 2 and 4, the anomaly mediation is dominant. Our models generalize those results. Since the parameters A and B can expand wider range in our models, the parameter α varies a wider parameter region, even if n_T and n_p are fixed. In model 3, we obtain a negative value of α , which would be naturally of $\mathcal{O}(1)$. In model 1, we have $0 < \alpha < 1$ when we fix $n_T = 3$ and $n_p = 2$. In addition, a value of α in model 2 and 4 is of $\mathcal{O}(4\pi^2)$. However, whether the modulus mediation and anomaly mediation are additive or destructive depends on gauge kinetic functions of visible sector. Suppose that the gauge kinetic function of the visible sector is given as

$$f_v = m_v S + w_v T, \quad (80)$$

where w_v can be positive and negative. Then, whether the modulus media-

tion and the anomaly mediation are additive or destructive depends on the signs of α and w_v . Furthermore, the size of gaugino masses induced by F^T is written as

$$M_{1/2}^{(T)} = \frac{F^T}{T + \bar{T}} \frac{1}{\left(\frac{m_v \langle ReS \rangle}{w_v \langle ReT \rangle} + 1\right)}. \quad (81)$$

Thus, it can be enhanced or suppressed. At any rate, soft SUSY breaking terms in our models have a rich structure. We will study their spectra and phenomenological aspects elsewhere.

Next, we discuss cosmological aspects of our models. The anomaly mediation is sizable except the special case with $|\alpha| \ll 1$. That implies that soft masses in the visible sector are suppressed by one-loop factor compared with the gravitino mass $m_{3/2}$, that is, the gravitino can be heavier like $m_{3/2} = \mathcal{O}(10)$ TeV. The modulus can have much larger mass. Therefore, we may avoid the gravitino problem and the moduli problem in all of models like the usual KKLT model [6, 9, 10, 11]. Furthermore, the potential forms of model 3 and 4 have more interesting aspects as discussed below.

The potential of model 1 and 2 has the runaway behavior at the right region of the bump like Figure 1, and the height of the bump is of $\mathcal{O}(m_{3/2}^2)$. When an initial value of T is in the right of the bump, T goes to infinity. Furthermore, when an initial value of T is quite small $T \ll 1$, T overshoots the favorable minimum and goes to infinity. Thus, we have to fine-tune the initial condition such that T is trapped at the favorable minimum. That is the overshooting problem [45].

This type of potential has another problem, that is, destabilization due to finite temperature effect. The finite temperature effect induces the additional potential term [46],

$$\Delta V = (\alpha_0 + \alpha_2 g^2) \hat{T}^4, \quad (82)$$

where \hat{T} denotes the temperature. The coefficients, α_0 and α_2 , are written by group factors of massless modes, and in most of case α_2 is positive. For example, we have $\alpha_2 = \frac{3}{8\pi^2}(N_c^2 - 1)(N_c + 3N_f)$ for SUSY $SU(N_c)$ gauge theory with N_f flavors of matter multiplets. In models 1 and 2, the gauge kinetic function is obtained as $f = mS + wT$. Thus, the potential term due to finite temperature effect is written as

$$\Delta V = \left[\alpha_0 + \alpha_2 \frac{1}{m \langle S \rangle + wT} \right] \hat{T}^4. \quad (83)$$

This term destabilizes T at not so high temperature, but the temperature corresponding to the intermediate scale [47].

The above two problems are not problems only for our models 1 and 2, but are rather generic problems. On the other hand, the potential in models 3 and 4 has the term like Be^{bT} . Such term may avoid the overshooting problem, and this term is reliable except when $Be^{bT} \geq \mathcal{O}(1)$, because in that region we would have uncontrollable effects. However, this reliable region of the potential is much higher than $\mathcal{O}(m_{3/2}^2)$, which is the height of the bump in model 1 and 2 as well as other potentials with this type of potential forms. The same behavior of the potential is helpful to avoid the destabilization problem due to finite temperature effects. For example, in model 3 the gauge kinetic function is written as $f = mS - wT$. Thus, the potential term due to finite temperature effects is written as

$$\Delta V = \left[\alpha_0 + \alpha_2 \frac{1}{m\langle S \rangle - wT} \right] \hat{T}^4. \quad (84)$$

That makes T shift to a smaller value, because smaller T corresponds to weaker coupling. Therefore, the VEV of T does not destabilize. Model 4 also has the same behavior. As results, the potential form in models 3 and 4 are cosmologically interesting from the viewpoint to avoid the overshooting problem and destabilization problem due to finite temperature.

6 Conclusion

We have considered the KKLT model with moduli-mixing superpotential, assuming that one of them is frozen. Such superpotential can be obtained e.g. by gluino condensation on magnetized D-branes, while it may be generated in other setups. We have studied four types of models. In these models, the hierarchy between the Planck scale and gravitino mass can be written by gauge coupling, such as $\ln(M_p/m_{3/2}) \simeq 8\pi^2 \langle Ref \rangle / N$, that is, magnetic fluxes can generate a large hierarchy. Model 1 is almost the same as the KKLT model, but the ratio α between anomaly mediation and modulus mediation can take various values. Models 2, 3, 4 do not require fine-tuning of 3-form fluxes to realize $\langle ReT \rangle \sim 1$ because of very small coefficient B . However, model 3 needs fine-tuning of 3-form fluxes in order to obtain a weak coupling on the magnetized D9-brane, but in the case that w_0 can be generated by gluino condensation on D3-brane, we may not need to tune minutely. We may need to tune slightly the open string sector such as magnetic fluxes,

the winding number and the number of D-branes instead of 3-form fluxes (in the closed string sector). In model 3 and 4, α becomes negative. All of models lead to a rich structure of SUSY breaking including new patterns of soft SUSY breaking terms. Such spectra and their phenomenological aspects would be studied elsewhere.

In most of models except for $|\alpha| \ll 1$, the gravitino and moduli masses are of $\mathcal{O}(10)$ TeV or much heavier. Such spectrum is important to avoid the gravitino problem and the moduli problem. Furthermore, the potential form of models 3 and 4 have good properties for cosmology because of the exponential factor, $\exp[+bT]$ for $b > 0$. That may avoid the overshooting problem and the destabilization problem due to finite temperature effects.

We have studied the models that two moduli fields S and T have mixing in superpotential, assuming one of them S is frozen already around the string scale M_{string} . Alternatively, both of them may remain light. In Ref. [22], we will study such models, i.e., moduli stabilization and SUSY breaking.

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A Analysis on the KKLT type scalar potential

In this appendix, we summarize analysis on the KKLT type scalar potential.

A.1 SUSY potential

First we consider the following F-term scalar potential V_F of supergravity model,

$$V_F = e^G \left[G_I G_{\bar{J}} G^{I\bar{J}} - 3 \right], \quad (85)$$

$$G = K + \ln |W|^2, \quad (86)$$

where $G_I = \partial_I G$, $G_{I\bar{J}} = \partial_I \partial_{\bar{J}} G$.

The first derivative of V_F is obtained as

$$\begin{aligned} \partial_I V_F &= e^G \left[G_I \cdot \left(G_K G_{\bar{L}} G^{K\bar{L}} - 2 \right) \right. \\ &\quad \left. + G_{\bar{L}} \cdot \left(G_{KI} G^{K\bar{L}} + G_K \left(\partial_I G^{K\bar{L}} \right) \right) \right], \end{aligned} \quad (87)$$

$$\begin{aligned} &= F^K \left[(\partial_I K_{K\bar{L}}) \bar{F}^{\bar{L}} + K_{K\bar{L}} \partial_I \bar{F}^{\bar{L}} \right] \\ &\quad + \bar{F}^{\bar{L}} \left[K_{K\bar{L}} \partial_I F^K + 3e^{K/2} \bar{W} K_{I\bar{L}} \right], \end{aligned} \quad (88)$$

where we have defined $F^I = -K^{I\bar{J}} e^{K/2} \bar{D}_{\bar{J}} \bar{W}$. Thus, the SUSY point, i.e. $G_I = D_I W = 0$, satisfies the stationary condition, $\partial_I V_F = 0$. At such SUSY point, mass matrices are given by

$$V_{IJ} = -e^G G_{IJ}, \quad (89)$$

$$V_{\bar{I}\bar{J}} = -e^G G_{\bar{I}\bar{J}}, \quad (90)$$

$$V_{I\bar{J}} = e^G \left[-2G_{I\bar{J}} + G_{IK} G_{\bar{J}\bar{L}} G^{K\bar{L}} \right]. \quad (91)$$

Here we concentrate to the model with only single field X . In this case, mass matrix is given as

$$\begin{pmatrix} V_{X\bar{X}} + \text{Re}(V_{XX}) & -\text{Im}(V_{XX}) \\ -\text{Im}(V_{XX}) & V_{X\bar{X}} - \text{Re}(V_{XX}) \end{pmatrix}, \quad (92)$$

in the basis of $(\text{Re}(x), \text{Im}(x))$, where $x = X - X_0$ with X_0 satisfying $\partial_X V|_{X=X_0} = 0$. Their eigenvalues are obtained as

$$V_{X\bar{X}} \pm |V_{XX}|. \quad (93)$$

Therefore, the SUSY point corresponds to the minimum of the potential if $V_{X\bar{X}} > |V_{XX}|$, that is,

$$|G_{XX}| > 2G_{X\bar{X}}. \quad (94)$$

A.2 KKLT type potential with uplifting potential

Now we consider the KKLT type potential. We use the following form of Kähler potential and generic form of superpotential,

$$K = -n_T \ln(T + \bar{T}), \quad n_T \in \mathbf{N}, \quad (95)$$

$$W = W(T). \quad (96)$$

The original KKLT model has $n_T = 3$. The SUSY point, i.e., $D_TW = 0$, is obtained as

$$(T + \bar{T}) = n_T \frac{W}{W_T} \in \mathbf{R}. \quad (97)$$

This point corresponds to the minimum of V_F if the VEV of T satisfies

$$\left| \frac{n_T(n_T - 1)}{(T + \bar{T})^2} - \frac{W_{TT}}{W} \right| > \frac{2n_T}{(T + \bar{T})^2}. \quad (98)$$

We consider the case that the above condition is satisfied, but such minimum of V_F is the SUSY AdS vacuum with the vacuum energy $V_F = -3e^K |W|^2 = -3m_{3/2}^2$.

In order to realize the dS vacuum, we add the following form of uplifting potential,

$$V_L = \frac{D}{(T + \bar{T})^{n_p}}, \quad 0 < D \ll 1. \quad (99)$$

In the original KKLT model, this is originated from $\overline{D}3$ -brane and we have $n_p = 2$. Here following Ref. [5, 6], we consider generic integer for n_p . Now the total potential is written as

$$V = V_F + V_L, \quad (100)$$

and adding V_L changes only vacuum value of ReT from (97) but not ImT . We demand that a change of ReT is small and the cosmological constant $\langle V \rangle$ vanishes, that is,

$$\langle V_L \rangle = -\langle V_F \rangle \simeq 3m_{3/2}^2. \quad (101)$$

From (88), the first derivative of V is written as

$$\begin{aligned} \partial_T V &= F^T \left[\frac{-2n_T}{(T + \bar{T})^3} \bar{F}^{\bar{T}} + \frac{n_T}{(T + \bar{T})^2} \partial_T \bar{F}^{\bar{T}} \right] \\ &+ \bar{F}^{\bar{T}} \left[\frac{n_T}{(T + \bar{T})^2} \partial_T F^T + 3e^{K/2} \bar{W} \frac{n_T}{(T + \bar{T})^2} \right] \\ &- \frac{n_p}{(T + \bar{T})} V_L. \end{aligned}$$

Now, we expand as $T = T_{SUSY} + \delta T$ such as $D_T W|_{T=T_{SUSY}} = 0$. Then, the F-term and its derivatives are evaluated as

$$\bar{F}^{\bar{T}} = e^{K/2} \frac{(T + \bar{T})}{n_T} (n_T W - (T + \bar{T}) W_T), \quad (102)$$

$$\partial_T \bar{F}^{\bar{T}} \simeq a(T + \bar{T}) \cdot e^{K/2} W = a(T + \bar{T}) m_{3/2}, \quad (103)$$

$$\partial_{\bar{T}} \bar{F}^{\bar{T}} \simeq -e^{K/2} W = -m_{3/2}. \quad (104)$$

Here we have used $W_T \simeq n_T W / (T + \bar{T})$ and defined a real parameter a ,

$$a \equiv -\frac{W_{TT}}{W_T}. \quad (105)$$

We have also assumed that $|a| \cdot \text{Re} T_{SUSY} \gg 1$. There is an important point for CP phase. Since

$$\delta T, \quad \frac{W_T}{W}, \quad \frac{W_{TT}}{W} \in \mathbf{R}, \quad (106)$$

the CP phase of W and $\bar{F}^{\bar{T}}$ are the same. That is, when we write $W = |W|e^{i\theta_W}$, then we have $\bar{F}^{\bar{T}} \propto e^{i\theta_W}$. Using the above results, we can write

$$\partial_T V \simeq m_{3/2} \frac{n_T}{(T + \bar{T})^2} \cdot \left[a(T + \bar{T}) F^T \right] - \frac{n_p}{(T + \bar{T})} V_L. \quad (107)$$

Therefore, for $a \text{Re} T \gg 1$ the stationary condition $\partial_T V = 0$ and the condition for the vanishing cosmological constant lead to

$$\frac{F^T}{(T + \bar{T})} \simeq n_p \frac{3}{n_T} \frac{m_{3/2}}{a(T + \bar{T})} e^{-i\theta_W}, \quad (108)$$

$$\frac{\delta T}{T_{SUSY}} \simeq \frac{n_p}{2} \frac{3}{n_T} \frac{1}{a^2 (\text{Re} T_{SUSY})^2} \ll 1. \quad (109)$$

Furthermore, the F-component of conformal compensator superfield ϕ is given by

$$\frac{F_\phi}{\phi_0} = e^{K/2} \bar{W} + \frac{1}{3} K_I F^I, \quad (110)$$

$$\simeq e^{-i\theta_W} m_{3/2}, \quad (111)$$

where $\phi_0 (\in \mathbf{R})$ is a scalar component of ϕ . Hence, CP phases of F_ϕ and F^T are aligned [6, 33]. It is useful to define the ratio α as follows,

$$\alpha \equiv \frac{F_\phi(T + \bar{T})}{\phi_0 \ln(M_p/m_{3/2}) F^T} = \frac{n_T a(T + \bar{T})}{3 n_p \ln(M_p/m_{3/2})}, \quad (112)$$

because of $\ln(M_p/m_{3/2}) \sim \mathcal{O}(4\pi^2)$.

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